AIAA JOURNAL
Vol. 42, No. 8, August 2004

Transient Analysis of Mode III Fracture in Anisotropic Layered Media

Yi-Shyong Ing* and Chung-Bin Chuang†
Tamkang University, Taipei 251, Taiwan, Republic of China

The transient response of a semi-infinite crack in an anisotropic layered medium is presented in this study. The individual layers are anisotropic and homogeneous. The number of reflections and diffractions of stress waves that are generated from the interfaces of the layered medium and the crack are infinite. It will cause extreme difficulties in analyzing this problem. A modified linear coordinate transformation is introduced to reduce the anisotropic layered medium problem to an equivalent isotropic problem with a similar geometry configuration. Besides, a useful fundamental solution is proposed and the solution can be determined by superposition of the fundamental solution in the Laplace transform domain. The Cagniard method of Laplace inversion is used to obtain the analytical transient solution in the time domain. The final results for the stress intensity factor are expressed in compact formulations and are valid for an infinite length of time. Numerical calculations are also evaluated and discussed in detail.

Nomenclature

b_j	=	slowness in anisotropic materials
$C_{44}^{(j)}, C_{45}^{(j)}, C_{55}^{(j)}$	=	elastic moduli
K	=	mode III dynamic stress intensity factor
t	=	time coordinate
$W^{(j)}$	=	antiplane displacement in corresponding isotropic layered media
$w^{(j)}$	=	antiplane displacement in anisotropic layered media
X, Y, Z	=	rectangular coordinates in corresponding isotropic layered media
x, y, z	=	rectangular coordinates in anisotropic layered media
$\rho^{(j)}$	=	material mass density
$\tau_{XZ}^{(j)}, \tau_{YZ}^{(j)}$	=	shear stresses in corresponding isotropic layered media
$\tau_{xz}^{(j)}, \tau_{yz}^{(j)}$	=	shear stresses in anisotropic layered media

Introduction

FOR the past two decades, the importance of composite materials has increased very rapidly because of their high strength and light weight. Due to the fast development in material science, anisotropic materials have been widely used in these composite structures for modern engineering applications such as aircrafts, vessels, nuclear plants, and so forth. These anisotropic laminated composites are usually subjected to dynamic loading. However, the study of dynamic fracture problems of anisotropic composite laminates is rare in the literature. This work is the extension of the study given by Ma and Ing[1] in which the attention was focused on the dynamic fracture analysis of an isotropic layered medium. In this study, an anisotropic layered medium containing a semi-infinite crack is considered. This investigation will give a basic understanding for studying dynamic fracture problems of anisotropic solids.

Recent developments in anisotropic elasticity were reviewed by Ting.[2] Ting[3] pointed out that there have been several new developments in the theory and applications of anisotropic elasticity. However, there were not many studies for dynamic fracture analysis of anisotropic solids in the literature. Most of the investigations on elastodynamic crack analysis of anisotropic solids were limited to transversely isotropic or orthotropic materials. In recent studies, Kundu and Boström[4] investigated the scattering of arbitrary elastic waves by a circular crack in a transversely isotropic solid. They calculated the crack opening displacements and the far field under time-harmonic loading. Zhang and Gross[5] discussed many elastodynamic problems of antiplane cracks in transversely isotropic solids. Zhang and Gross[5] also gave a detailed list of references to anisotropic dynamic fracture problems. Rizza and Nair[6] used a numerical method to solve problems involving nonaxisymmetric dynamic impact loading of a penny-shaped crack in a transversely isotropic medium. A three-dimensional problem for a transversely isotropic crack subjected to dynamic concentrated point forces has been analyzed by Zhao.[7] Sarkar et al.[8] have analyzed the dynamic response of three coplanar Griffith cracks in an orthotropic medium. They used the Fourier and finite Hilbert transform techniques to obtain approximate values of the crack opening displacements and stress intensity factors. The time-harmonic problem of two parallel cracks in an infinite orthotropic plane was studied by Itou and Haliding.[9] They have calculated the dynamic stress intensity factors numerically for composite materials. Das and Patra[10,11] have investigated the plane strain problem of dynamic stress intensity factors for a moving Griffith crack situated at the interface of two dissimilar orthotropic fixed layers and half-planes. The authors solved the reduced singular integral equations by using Jacobi's polynomials. Finite cracks in orthotropic materials under dynamic loadings have been investigated by Kassir and Bandyopadhyay,[12] Shindo and Nozaki,[13] and Rubio-Gonzalez and Mason[14,15] using the integral transform method. This method leads to a Fredholm integral equation rather than a Wiener–Hopf equation on the Laplace transform domain. They solved the Fredholm integral equation in the Laplace transform domain numerically, and the dynamic stress intensity factor was obtained by numerical Laplace inversion. Rubio-Gonzalez and Mason[16] obtained analytical closed-form solutions for dynamic stress intensity factors of a semi-infinite crack under uniform crack face loading in orthotropic materials. Brock and Hanson[17] studied a transient plane strain analysis of diffraction of plane waves by a semi-infinite crack in an unbounded orthotropic or transversely isotropic solid. Zhang[18] applied the time-domain traction boundary integral equation to obtain the dynamic stress intensity factor of a finite crack in anisotropic solids under antiplane deformation. The hypersingular integral equation method for anisotropic crack scattering and application to modeling of ultrasonic nondestructive evaluation was reviewed by Boström.[19] Zhang[18] and

Received 7 July 2003; revision received 30 November 2003; accepted for publication 7 January 2004.

*Associate Professor, Department of Aerospace Engineering, Tamsui; ysing@mail.tku.edu.tw.
†Graduate Student, Department of Aerospace Engineering, Tamsui.

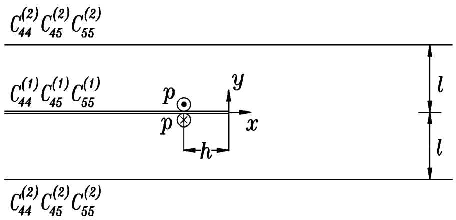

Fig. 1 Geometry of a semi-infinite crack in an anisotropic layered medium.

Boström[19] also gave detailed reviews of dynamic anisotropic crack problems.

For anisotropic interface crack problems, Shen and Kuang[20] investigated the problem of wave scattering from an interface crack in laminated anisotropic media. In their study, the wave fields were obtained numerically. Pramanik et al.[21] have analyzed the transient elastodynamic problem of a Griffith crack lying at the interface of two dissimilar anisotropic half-planes. The results yield generalized Wiener–Hopf-type equations and the arising integral equations are solved by the standard iteration technique. Ma and Liao[22] studied the transient full-field response of a semi-infinite interface crack lying between dissimilar anisotropic media subjected to a dynamic body force.

Because of the mathematical difficulties, the explicitly analytical solution for the problem of a semi-infinite crack subjected to dynamic point loading in anisotropic layered media has not yet been obtained. It is important to develop a mathematical method to obtain solutions for cracks in an anisotropic layered medium. In this study, the theoretical transient analysis is performed for a semifinite crack in an anisotropic three-layered medium subjected to an antiplane concentrated dynamic loading as shown in Fig. 1. Although this boundary condition is chiefly a mathematically convenient proposition, the obtained transient solution can give a basic understanding of physically transient problems. Moreover, because the transient solution is exact up to the arrival time of the next wave, this boundary condition is also appropriate in the early transient stage if other dimensions of the structure are much greater than the height of the middle layer and the distance from applied loading to the crack tip. A modified linear coordinate transformation is proposed in this study to simplify the anisotropic problem. The linear coordinate transformation reduces the anisotropic layered problem to an equivalent isotropic problem. The number of reflections and diffractions of stress waves that are generated from the interfaces of the layered medium and the crack is infinite. It will cause extreme difficulties and it is impossible to solve this complicated problem by using the standard integral transform method. A useful fundamental solution is proposed to overcome these difficulties. Similar fundamental solutions have been used by Ing and Ma[23–26] to solve isotropic and anisotropic finite crack problems. The Cagniard method[27] of Laplace inversion is used to obtain the transient solution in the time domain. The final results for dynamic stress intensity factors are expressed in compact formulations.

Modified Linear Coordinate Transformation

Consider the problem of a semi-infinite crack in an infinitely long strip with a finite width of $2l$ that is sandwiched between two half-planes as shown in Fig. 1. Each layer is made of a homogeneous, anisotropic, linearly elastic material. In the absence of body forces, the two-dimensional antiplane wave motion of each anisotropic layer in terms of the displacement is governed by

$$C_{55}^{(j)} \frac{\partial^2 w^{(j)}(x,y,t)}{\partial x^2} + 2C_{45}^{(j)} \frac{\partial^2 w^{(j)}(x,y,t)}{\partial x \partial y} + C_{44}^{(j)} \frac{\partial^2 w^{(j)}(x,y,t)}{\partial y^2} = \rho^{(j)} \frac{\partial^2 w^{(j)}(x,y,t)}{\partial t^2}, \qquad j = 1,2 \quad (1)$$

where $w^{(j)}(x,y,t)$ is the displacement in the z direction, $C_{kl}^{(j)}(k,l=4,5)$ is the elastic modulus, and $\rho^{(j)}$ is the mass density of the anisotropic material. The superscript $j=1$ represents the quantities related to the middle layer (material 1), whereas $j=2$ is for the lower and upper half-planes (material 2). The x–y plane has been assumed to coincide with one of the planes of material symmetry so that in-plane and antiplane deformations are uncoupled. The relevant shear stress components are

$$\tau_{yz}^{(j)}(x,y,t) = C_{44}^{(j)} \frac{\partial w^{(j)}(x,y,t)}{\partial y} + C_{45}^{(j)} \frac{\partial w^{(j)}(x,y,t)}{\partial x} \qquad j = 1,2 \quad (2)$$

$$\tau_{xz}^{(j)}(x,y,t) = C_{45}^{(j)} \frac{\partial w^{(j)}(x,y,t)}{\partial y} + C_{55}^{(j)} \frac{\partial w^{(j)}(x,y,t)}{\partial x} \qquad j = 1,2 \quad (3)$$

although there were some linear coordinate transformations in the literature.[28–30] To solve the complicated problem of the anisotropic layered medium, a modified linear coordinate transformation is introduced as follows:

For $y \ge 0$,

$$X = x + \alpha^{(j)} y + (j-1)\cdot l \cdot \left(\alpha^{(1)} - \alpha^{(2)}\right), \qquad j = 1,2 \quad (4)$$

$$Y = \beta^{(j)} y + (j-1)\cdot l \cdot \left(\beta^{(1)} - \beta^{(2)}\right), \qquad j = 1,2 \quad (5)$$

$$Z = z \quad (6)$$

For $y \le 0$,

$$X = x + \alpha^{(j)} y + (j-1)\cdot l \cdot \left(\alpha^{(2)} - \alpha^{(1)}\right), \qquad j = 1,2 \quad (7)$$

$$Y = \beta^{(j)} y + (j-1)\cdot l \cdot \left(\beta^{(2)} - \beta^{(1)}\right), \qquad j = 1,2 \quad (8)$$

$$Z = z \quad (9)$$

where

$$\alpha^{(j)} = -C_{45}^{(j)} \Big/ C_{44}^{(j)} \quad (10)$$

$$\beta^{(j)} = C_e^{(j)} \Big/ C_{44}^{(j)} \quad (11)$$

$$C_e^{(j)} = \sqrt{C_{44}^{(j)} C_{55}^{(j)} - C_{45}^{(j)^2}} \quad (12)$$

Assume that $C_{44}^{(j)}$ and $C_{55}^{(j)}$ as well as $\sqrt{(C_{44}^{(j)} C_{55}^{(j)} - C_{45}^{(j)^2})}$ are all positive to ensure the positive definite of strain energy. The transformation given by Eqs. (4–9) reduces Eq. (1) to the standard antiplane wave equation for the isotropic solid in the (X,Y) coordinate system as

$$\frac{\partial^2 W^{(j)}(X,Y,t)}{\partial X^2} + \frac{\partial^2 W^{(j)}(X,Y,t)}{\partial Y^2} = b_j^2 \frac{\partial^2 W^{(j)}(X,Y,t)}{\partial t^2} \qquad j = 1,2 \quad (13)$$

where $W^{(j)}(X,Y,t)$ is the displacement in the Z direction and

$$W^{(j)}(X,Y,t) = w^{(j)}(x,y,t) \quad (14)$$

$$b_j = \sqrt{C_{44}^{(j)} \rho^{(j)}} \Big/ C_e^{(j)} \quad (15)$$

The stress components expressed in Eqs. (2) and (3) for the anisotropic solid are related to those in the corresponding isotropic solid by

$$\tau_{YZ}^{(j)}(X,Y,t) = C_e^{(j)} \frac{\partial W^{(j)}(X,Y,t)}{\partial Y} \quad (16)$$

$$\tau_{XZ}^{(j)}(X,Y,t) = C_e^{(j)} \frac{\partial W^{(j)}(X,Y,t)}{\partial X} \quad (17)$$

$$\tau_{yz}^{(j)}(x,y,t) = \tau_{YZ}^{(j)}(X,Y,t) \quad (18)$$

$$\tau_{xz}^{(j)}(x,y,t) = \frac{C_{45}^{(j)}}{C_{44}^{(j)}} \tau_{YZ}^{(j)}(X,Y,t) + \frac{C_e^{(j)}}{C_{44}^{(j)}} \tau_{XZ}^{(j)}(X,Y,t) \quad (19)$$

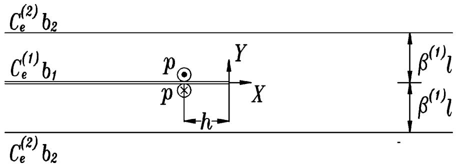

Fig. 2 Geometry of the cracked layered medium after linear coordinate transformation.

It can be verified that the interface boundaries between the middle strip and two half-planes remain continuous after the transformation. The transformed configuration is plotted in Fig. 2. Several characteristics are identified here. First, the interfaces remain straight, continuous, and parallel to each other; that is, no gaps or overlaps are generated along the interfaces. Second, the thickness of the middle strip becomes $2\beta^{(1)}l$ instead of $2l$. Finally, the X axis is coincident with the x axis even after the transformation, which means that the crack still lies on the negative X axis after the transformation.

From Eqs. (13), (16), and (17), it is noted that the original anisotropic layered medium problem is converted into an equivalent isotropic layered medium problem by setting $C_e^{(j)} = \mu^{(j)}$ (shear modulus). From the relationship of displacement and shear stresses for an anisotropic solid and the correspondent isotropic solid expressed in Eqs. (14), (18), and (19), one can see that it is possible to obtain the solution for an anisotropic problem from a correspondent result of an isotropic problem.

Required Fundamental Solutions

A useful fundamental problem is proposed here and the corresponding fundamental solutions are applied to solve the complicated problem in the next section. Consider an unbounded anisotropic material (indicated as material 1) containing a semi-infinite crack lying on the negative x axis. An antiplane exponentially distributed traction in the Laplace transform domain is applied on the upper and lower crack faces. The boundary conditions on crack surfaces expressed in the Laplace transform domain for this fundamental problem are represented as follows:

$$\bar{\tau}_{yz}^{(1)}(x, 0, s) = e^{s\eta x} \qquad \text{for} \qquad -\infty < x < 0 \tag{20}$$

$$\bar{w}^{(1)}(x, 0, s) = 0 \qquad \text{for} \qquad 0 < x < \infty \tag{21}$$

where s is the Laplace transform parameter and η is a constant. The overbar symbol is used for denoting the transform on time t. By using Eqs. (4–9), (14), and (18), the boundary conditions in Eqs. (20) and (21) can be rewritten as

$$\bar{\tau}_{YZ}^{(1)}(X, 0, s) = e^{s\eta X} \qquad \text{for} \qquad -\infty < X < 0 \tag{22}$$

$$\bar{W}^{(1)}(X, 0, s) = 0 \qquad \text{for} \qquad 0 < X < \infty \tag{23}$$

The fundamental problem for governing equation (13) (setting $j = 1$) subjected to boundary conditions (22) and (23) can be solved by application of the standard integral transform method. The one-sided Laplace transform with respect to time t and the two-sided Laplace transform with respect to the spatial coordinate X are defined by[31]

$$\bar{f}(X, Y, s) = \int_0^{\infty} f(X, Y, t)\, e^{-st}\, \mathrm{d}t \tag{24}$$

$$\bar{f}^{*}(\lambda, Y, s) = \int_{-\infty}^{\infty} \bar{f}(X, Y, s) e^{-s\lambda X}\, \mathrm{d}X \tag{25}$$

Applying the one-sided Laplace transform over time, and the two-sided Laplace transform over X under the restriction of $\mathrm{Re}(\eta) > \mathrm{Re}(\lambda)$, and the Wiener–Hopf technique is finally implemented. The solutions of shear stresses and displacement in the Laplace transform domain can be expressed as follows:

$$\bar{\tau}_{YZ}^{(1)}(X, Y, s) = \frac{1}{2\pi i} \int_{\Gamma_\lambda} \frac{\gamma_{1+}(\lambda) e^{-s[\gamma_1(\lambda)|Y| - \lambda X]}}{\gamma_{1+}(\eta)(\eta - \lambda)}\, \mathrm{d}\lambda \tag{26}$$

$$\bar{\tau}_{XZ}^{(1)}(X, Y, s) = \frac{-\mathrm{sign}(Y)}{2\pi i} \int_{\Gamma_\lambda} \frac{\lambda e^{-s[\gamma_1(\lambda)|Y| - \lambda X]}}{\gamma_{1+}(\eta)(\eta - \lambda)\gamma_{1-}(\lambda)}\, \mathrm{d}\lambda \tag{27}$$

$$\bar{W}^{(1)}(X, Y, s) = \frac{-\mathrm{sign}(Y)}{2\pi i} \int_{\Gamma_\lambda} \frac{e^{-s[\gamma_1(\lambda)|Y| - \lambda X]}}{C_e^{(1)} s \gamma_{1+}(\eta)(\eta - \lambda)\gamma_{1-}(\lambda)}\, \mathrm{d}\lambda \tag{28}$$

where Γ_λ is the path of integration in the complex λ plane and

$$\mathrm{sign}(Y) = \begin{cases} 1 & \text{if} \quad Y \geq 0^+ \\ -1 & \text{if} \quad Y \leq 0^- \end{cases}$$

$$\gamma_{1+}(\lambda) = \sqrt{b_1 + \lambda}, \qquad \gamma_{1-}(\lambda) = \sqrt{b_1 - \lambda}$$

$$\gamma_1(\lambda) = \gamma_{1+}(\lambda) \cdot \gamma_{1-}(\lambda)$$

The corresponding result of the dynamic stress intensity factor in the Laplace transform domain is

$$\bar{K}(s) = \lim_{x \to 0^+} \sqrt{2\pi X}\, \bar{\tau}_{YZ}^{(1)}(X, 0, s) = -\sqrt{2} \Big/ \sqrt{s}\gamma_{1+}(\eta) \tag{29}$$

Since $\tau_{yz}^{(1)}(x, y, t) = \tau_{YZ}^{(1)}(X, Y, t)$, as indicated in Eq. (18), the stress intensity factor in the transformed coordinates (X, Y, Z) is exactly the same as that in the original coordinates (x, y, z).

Transient Analysis of Dynamic Stress Intensity Factors

Consider a semi-infinite crack in an anisotropic strip with a height of $2l$ that is sandwiched between two anisotropic half-planes as shown in Fig. 1. The strip and half-planes are made of anisotropic materials 1 and 2, respectively. The semi-infinite crack lies in the middle of the strip and parallel to the interfaces. For time $t < 0$, the cracked composite is stress-free and at rest. At $t = 0$, a pair of dynamic concentrated loadings with magnitude p is applied at the crack faces with a distance h from the crack tip. The time dependence of the loading is represented by the Heaviside step function $H(t)$. From Eq. (1), the governing equation for the anisotropic strip can be expressed as follows:

$$C_{55}^{(1)} \frac{\partial^2 w^{(1)}(x, y, t)}{\partial x^2} + 2C_{45}^{(1)} \frac{\partial^2 w^{(1)}(x, y, t)}{\partial x \partial y} + C_{44}^{(1)} \frac{\partial^2 w^{(1)}(x, y, t)}{\partial y^2} = \rho^{(1)} \frac{\partial^2 w^{(1)}(x, y, t)}{\partial t^2} \tag{30}$$

The boundary condition for the applied loading can be represented by

$$\tau_{yz}^{(1)}(x, 0, t) = -p\delta(x - h)H(t) \tag{31}$$

where $\delta()$ is the Dirac delta function.

A linear coordinate transformation proposed in the previous section is used here to solve the anisotropic layered medium problem. By using the linear coordinate transformation, the corresponding isotropic configuration is illustrated in Fig. 2. It is noted that the width of the strip is changed from $2l$ to $2\beta^{(1)}l$, whereas the location of the concentrated loading is unchanged. Relations between the anisotropic problem and the corresponding isotropic problem have been established and discussed in the preceding section. Solutions for the anisotropic layered medium problem can be obtained from the correspondent results of an isotropic problem.

The major difficulty in analyzing this problem is that the interaction of reflected waves with the semi-infinite crack should be taken into account. A methodology of superposition of the fundamental solution in the Laplace transform domain is used to overcome this difficulty. It is noticed that the theoretical analysis performed in this study is valid only for the case where $b_1 < b_2$. The solutions are composed of incident, reflected, and diffracted fields, which are denoted by superscripts of i, r, and d, respectively. Before the time that i and d waves are reflected by the interfaces, it can be treated as a problem containing a semi-infinite crack in an infinite solid. After a linear coordinate transformation, the incident field of the cylindrical wave generated by the concentrated loading in Eq. (31) can be expressed in the Laplace transform domain as follows:

$$\bar{\tau}_{YZ}^{(1),i}(X,0,s) = \frac{-1}{2\pi i}\int_{\Gamma_\lambda} p e^{s\lambda(X+h)}\,\mathrm{d}\lambda \tag{32}$$

The applied traction on the crack faces, as indicated in Eq. (32), has a functional form $e^{s\lambda X}$. Because the solutions of applying traction $e^{s\eta X}$ on crack faces have been obtained previously, the diffracted field can be constructed by superimposing the incident wave traction that is equal to Eq. (32). Because of the geometric symmetry, only $Y \geq 0$ will be considered. By combining Eqs. (26) and (32), the incident and diffracted fields in the Laplace transform domain for $Y \geq 0$ can be obtained as follows:

$$\bar{\tau}_{YZ}^{(1),i+d}(X,Y,s) = \frac{-1}{2\pi i}\int_{\Gamma_{\eta_1}} p\left\{\frac{1}{2\pi i}\int_{\Gamma_{\eta_2}} \frac{\gamma_{1+}(\eta_2)}{\gamma_{1+}(\eta_1)(\eta_1-\eta_2)} \times e^{s\eta_1 h} e^{-s[\gamma_1(\eta_2)Y-\eta_2 X]}\,\mathrm{d}\eta_2\right\}\mathrm{d}\eta_1 \tag{33}$$

By using the Cagniard method of Laplace inversion, the stress fields in time domain can be obtained as follows:

$$\tau_{YZ}^{(1),i}(X,Y,t) = \frac{-pt\sin\psi_i}{\pi R_i\left(t^2-b_1^2R_i^2\right)^{\frac{1}{2}}}H(t-b_1R_i) \tag{34}$$

$$\tau_{YZ}^{(1),d}(X,Y,t) = \frac{p}{2\pi^2}\int_{b_1h}^{t-b_1R_d}\mathrm{Re}\left[G\left(\eta_1^+,\eta_2^+\right)\frac{\partial\eta_1^+}{\partial t_1}\frac{\partial\eta_2^+}{\partial t_2} - G\left(\eta_1^-,\eta_2^+\right)\frac{\partial\eta_1^-}{\partial t_1}\frac{\partial\eta_2^+}{\partial t_2}\right]\mathrm{d}t_1 \tag{35}$$

where

$$\eta_1^{\pm} = -\frac{t_1}{h} \pm i\varepsilon, \qquad \eta_2^{\pm} = -\frac{t_2}{R_d}\cos\psi_d \pm i\frac{\sin\psi_d}{R_d^2}\left(t_2^2 - b_1^2R_d^2\right)^{\frac{1}{2}}$$

$$R_i = [(X+h)^2+Y^2]^{\frac{1}{2}}, \qquad \psi_i = \cos^{-1}\left(\frac{X+h}{R_i}\right)$$

$$R_d = (X^2+Y^2)^{\frac{1}{2}}, \qquad \psi_d = \cos^{-1}\left(\frac{X}{R_d}\right)$$

$$G(\eta_1,\eta_2) = \frac{\gamma_{1+}(\eta_2)}{\gamma_{1+}(\eta_1)(\eta_1-\eta_2)}, \qquad t = t_1+t_2$$

The corresponding stress intensity factor expressed in the Laplace transform domain is

$$\bar{K}^d(s) = \frac{-1}{2\pi i}\int_{\Gamma_\lambda} p e^{s\lambda h}\left[\frac{-\sqrt{2}}{\sqrt{s}\gamma_{1+}(\lambda)}\right]\mathrm{d}\lambda \tag{36}$$

The dynamic stress intensity factor that is induced by the diffracted d wave in the time domain is

$$K^d(t) = K^0(t) = p\sqrt{2/\pi h}\,H(t-b_1h) \tag{37}$$

The dynamic stress intensity factor shown in Eq. (37) jumps from zero to a constant value at the instance that the cylindrical wave generated by the concentrated loading reaches the crack tip. This solution is the same as that for a semi-infinite crack in an unbounded anisotropic medium subjected to a pair of dynamic concentrated loadings. In the following derivation, it is assumed that the dynamic stress intensity factor does not exceed its critical value, so the crack remains stationary. This assumption will be adopted throughout the whole paper.

The incident wave (i wave) generated from the dynamic concentrated loading and the diffracted wave (d wave) radiated out from the crack tip will be reflected by the interfaces after some later time. These reflected waves will be indicated as ir and dr waves, respectively. The waves reflected by the upper interface expressed in the Laplace transform domain can be obtained by the generalized ray method as follows:

$$\bar{\tau}_{YZ}^{(1),\mathrm{ir+dr}}(X,Y,s) = \frac{-1}{4\pi^2}\int_{\Gamma_{\eta_1}}\int_{\Gamma_{\eta_2}} \frac{pr_{\frac{1}{2}}(\eta_2)\gamma_{1+}(\eta_2)}{\gamma_{1+}(\eta_1)(\eta_1-\eta_2)} \times e^{s\eta_1 h}e^{s[\gamma_1(\eta_2)(Y-2\beta^{(1)}l)+s\eta_2X]}\,\mathrm{d}\eta_2\,\mathrm{d}\eta_1 \tag{38}$$

where

$$r_{\frac{1}{2}}(\eta_2) = \frac{C_e^{(1)}\gamma_1(\eta_2) - C_e^{(2)}\gamma_2(\eta_2)}{C_e^{(1)}\gamma_1(\eta_2) + C_e^{(2)}\gamma_2(\eta_2)} \tag{39}$$

is the reflection coefficient and $\gamma_2(\eta_2) = \sqrt{(b_2^2-\eta_2^2)}$.

Because the stress intensity factor is the key parameter in characterizing dynamic crack growth, we will focus our attention mainly on the determination of the dynamic stress intensity factor. By setting $Y = 0$ in Eq. (38) and using the fundamental solution in Eq. (29), the dynamic stress intensity factors that are induced by the reflected ir and dr waves have the following form:

$$\bar{K}^{\mathrm{ird+drd}}(s) = \frac{-1}{4\pi^2}\int_{\Gamma_{\eta_1}}\int_{\Gamma_{\eta_2}} \frac{p\sqrt{2}r_{\frac{1}{2}}(\eta_2)}{\sqrt{s}\gamma_{1+}(\eta_1)(\eta_1-\eta_2)} \times e^{s\eta_1 h}e^{-2s\gamma_1(\eta_2)\beta^{(1)}l}\,\mathrm{d}\eta_2\,\mathrm{d}\eta_1 \tag{40}$$

The dynamic stress intensity factors expressed in the time domain will be

$$K^{\mathrm{ird}}(t) = K^1(t) = \frac{\sqrt{2}p}{\pi^{\frac{3}{2}}}\int_{b_1r_1}^{t}\frac{1}{\sqrt{t-\tau}} \times \mathrm{Im}\left[\frac{r_{\frac{1}{2}}\left(\eta_{1,1}^+\right)\left(\partial\eta_{1,1}^+/\partial t_1\right)}{\gamma_{1+}\left(\eta_{1,1}^+\right)}\right]_{t_1=\tau}\mathrm{d}\tau \tag{41}$$

$$K^{\mathrm{drd}}(t) = K^{0,1}(t) = \frac{-\sqrt{2}p}{\pi^{\frac{5}{2}}}\int_{b_1h+2b_1\beta^{(1)}l}^{t}\int_{b_1h}^{\tau-2b_1\beta^{(1)}l}\frac{1}{\sqrt{t-\tau}} \times \mathrm{Re}\left[\frac{r_{\frac{1}{2}}\left(\eta_2^+\right)}{\gamma_{1+}\left(\eta_{1,0}^+\right)\left(\eta_{1,0}^+-\eta_2^+\right)}\frac{\partial\eta_{1,0}^+}{\partial t_1}\frac{\partial\eta_2^+}{\partial t_2}\right]_{t=\tau}\mathrm{d}t_1\,\mathrm{d}\tau \tag{42}$$

where

$$\eta_{1,1}^+ = t_1\cos\theta_1/r_1 + i(\sin\theta_1/r_1)\left(t_1^2-b_1^2r_1^2\right)^{\frac{1}{2}}$$

$$r_1 = \left[h^2+\left(2\beta^{(1)}l\right)^2\right]^{\frac{1}{2}}, \qquad \theta_1 = \cos^{-1}(-h/r_1)$$

$$\eta_{1,0}^+ = -t_1/h + i\varepsilon, \qquad \eta_2^+ = i\sqrt{\left(t_2/2\beta^{(1)}l\right)^2 - b_1^2}$$

$$t_1 + t_2 = t$$

Similarly, the irr, ird, and drd waves for shear stresses can be constructed by using Eq. (38) and the fundamental solution in Eq. (26). The final results yield

$$\bar{\tau}_{YZ}^{(1),\mathrm{irr+ird+drd}}(X,Y,s) = \frac{-i}{8\pi^3}\int_{\Gamma_{\eta_1}}\int_{\Gamma_{\eta_2}}\int_{\Gamma_{\eta_3}} \frac{pr_{\frac{1}{2}}(\eta_2)\gamma_{1+}(\eta_3)}{\gamma_{1+}(\eta_1)(\eta_1-\eta_2)(\eta_2-\eta_3)} \times e^{s\eta_1h-2s\gamma_1(\eta_2)\beta^{(1)}l}e^{-\gamma_1(\eta_3)Y+s\eta_3X}\,\mathrm{d}\eta_3\,\mathrm{d}\eta_2\,\mathrm{d}\eta_1 \tag{43}$$

After some later time, the irr, ird, and drd waves will be reflected by the interfaces again. The corresponding reflected waves are denoted by irrr, irdr, and drdr waves. By superimposing the fundamental solution in Eq. (29) with these reflected waves, the dynamic stress intensity factors induced by irrrd, irdrd, and drdrd waves can be obtained. Following a similar procedure, the complete solutions for the dynamic stress intensity factor of the cracked anisotropic layered medium that account for the contributions of all the reflected and diffracted waves are derived explicitly. The solutions can be simplified into a very compact form as follows:

$$K(t) = K^0(t) + 2\cdot\sum_{m=1}^{\infty} K^m(t) + \sum_{n=1}^{\infty} 2^n \cdot K^{0,n}(t) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} 2^{n+1}\cdot K^{m,n}(t) \tag{44}$$

where

$$K^0(t) = p\sqrt{\frac{2}{\pi h}}H(t - b_1 h)$$

$$K^m(t) = \frac{\sqrt{2}p}{\pi^{\frac{3}{2}}}\int_{b_1 r_m}^{t}\frac{1}{\sqrt{t-\tau}} \times \mathrm{Im}\left[\frac{r_{\frac{1}{2}}^m\left(\eta_{1,m}^{+}\right)\left(\partial\eta_{1,m}^{+}/\partial t_1\right)}{\gamma_{1+}\left(\eta_{1,m}^{+}\right)}\right]_{t_1=\tau}\mathrm{d}\tau$$

$$K^{m,n}(t) = \frac{2\sqrt{2}pi^q}{\sqrt{\pi}(2\pi i)^{n+1}}\int_{b_1 r_m + 2nb_1\beta^{(1)}l}^{t}\int_{b_1 r_m}^{a_1}\int_{2b_1\beta^{(1)}l}^{a_2} \times \int_{2b_1\beta^{(1)}l}^{a_n}\frac{1}{\sqrt{t-\tau}}SIF_{m,n}\,\mathrm{d}t_n\,\mathrm{d}_{t_{n-1}}\cdots\mathrm{d}t_1\,\mathrm{d}\tau$$

in which

$$a_1 = \tau - 2nb_1\beta^{(1)}l$$

$$a_j = \tau - t_1 - t_2 - t_3\cdots - t_{j-1} - 2(n - j + 1)b_1\beta^{(1)}l, \qquad j = 2,3,4,\ldots,n$$

$q = 0$ when $n = 1,3,5,\ldots$; $q = 1$, when $n = 2,4,6,\ldots$; and

$$SIF_{m,n} = \mathrm{Re}\left\{\frac{\left[r_{\frac{1}{2}}\left(\eta_{1,m}^{+}\right)\right]^m r_{\frac{1}{2}}\left(\eta_2^{\pm}\right)r_{\frac{1}{2}}\left(\eta_3^{\pm}\right)\cdots r_{\frac{1}{2}}\left(\eta_{n+1}^{\pm}\right)\left(\pm\partial\eta_{1,m}^{\pm}/\partial t_1\right)\left(\pm\partial\eta_2^{\pm}/\partial t_2\right)\cdots\left(\pm\partial\eta_n^{\pm}/\partial t_n\right)\left(\pm\partial\eta_{n+1}^{+}/\partial t_{n+1}\right)}{\gamma_{1+}\left(\eta_{1,m}^{\pm}\right)\left(\eta_{1,m}^{\pm}-\eta_2^{\pm}\right)\left(\eta_2^{\pm}-\eta_3^{\pm}\right)\cdots\left(\eta_{n-1}^{\pm}-\eta_n^{\pm}\right)\left(\eta_n^{\pm}-\eta_{n+1}^{+}\right)}\right\}_{t=\tau} \quad \text{for} \quad n = 1,3,5,\ldots$$

$$SIF_{m,n} = \mathrm{Im}\left\{\frac{\left[r_{\frac{1}{2}}\left(\eta_{1,m}^{+}\right)\right]^m r_{\frac{1}{2}}\left(\eta_2^{\pm}\right)r_{\frac{1}{2}}\left(\eta_3^{\pm}\right)\cdots r_{\frac{1}{2}}\left(\eta_{n+1}^{\pm}\right)\left(\pm\partial\eta_{1,m}^{\pm}/\partial t_1\right)\left(\pm\partial\eta_2^{\pm}/\partial t_2\right)\cdots\left(\pm\partial\eta_n^{\pm}/\partial t_n\right)\left(\pm\partial\eta_{n+1}^{+}/\partial t_{n+1}\right)}{\gamma_{1+}\left(\eta_{1,m}^{\pm}\right)\left(\eta_{1,m}^{\pm}-\eta_2^{\pm}\right)\left(\eta_2^{\pm}-\eta_3^{\pm}\right)\cdots\left(\eta_{n-1}^{\pm}-\eta_n^{\pm}\right)\left(\eta_n^{\pm}-\eta_{n+1}^{+}\right)}\right\}_{t=\tau} \quad \text{for} \quad n = 2,4,6,\ldots$$

$$\eta_{1,m}^{\pm} = \frac{t_1\cos\theta_m}{r_m} \pm i\frac{\sin\theta_m}{r_m}\left(t_1^2 - b_1^2 r_m^2\right)^{\frac{1}{2}}, \qquad r_m = \left[h^2 + \left(2m\beta^{(1)}l\right)^2\right]^{\frac{1}{2}}, \qquad \theta_m = \cos^{-1}\left(\frac{-h}{r_m}\right)$$

$$\eta_j^{\pm} = \pm i\sqrt{\left(\frac{t_j}{2\beta^{(1)}l}\right)^2 - b_1^2}, \qquad j = 2,3,4,\ldots$$

$$t_j = t - t_1 - t_2 - t_3\cdots - t_{j-1}, \qquad j = 2,3,4,\ldots$$

$$r_{\frac{1}{2}}(\lambda) = \frac{C_e^{(1)}\gamma_1(\lambda) - C_e^{(2)}\gamma_2(\lambda)}{C_e^{(1)}\gamma_1(\lambda) + C_e^{(2)}\gamma_2(\lambda)}$$

$$\gamma_{j+}(\lambda) = \sqrt{b_j + \lambda}, \qquad \gamma_{j-}(\lambda) = \sqrt{b_j - \lambda}, \qquad \gamma_j(\lambda) = \gamma_{j+}(\lambda)\cdot\gamma_{j-}(\lambda)$$

Numerical Results

In the preceding section, dynamic stress intensity factors of a semi-infinite crack subjected to dynamic antiplane concentrated loading in an anisotropic layered medium are derived and expressed in explicit forms. It is assumed that the stress intensity factor does not exceed the fracture toughness of the layered medium, and so the crack remains stationary all of the time. Because the transient solution is exact up to the arrival time of the next wave, only a finite number of waves will be involved in the numerical calculation. However, the numerical calculation includes many high-dimensional integrals; the convergence of the integration should be carefully examined. The program code splits the interval of integration into m ($m = 1, 2, 4, 8, 16, \ldots,$) segments; in each segment the integral is calculated by the 24-term Gaussian formula. If the two schemes give results close to each other, the value is accepted as the integrated value. Otherwise, the segment is subdivided into smaller regions and the process is repeated over these smaller segments. The results showed that the convergences of most cases can be achieved for $m = 2 \sim 8$. According to the previous definitions, the parameter $\beta^{(1)} = 1.0$ implies an isotropic case, whereas $\beta^{(1)} \neq 1.0$ indicates an anisotropic problem. If the material constant $C_e^{(2)} = 0$, the layered medium problem will degenerate to a single strip problem. The appropriate static solutions for a semi-infinite crack in an isotropic single strip[32] and an anisotropic single strip[28] are

$$K^{\mathrm{si}} = p\sqrt{2/l}\left(1 - e^{-\pi h/l}\right)^{-\frac{1}{2}} \tag{45}$$

$$K^{\mathrm{sa}} = p\sqrt{2C_{44}^{(1)}/C_e^{(1)}l}\left(1 - e^{-\pi C_{44}^{(1)}h/C_e^{(1)}l}\right)^{-\frac{1}{2}} \tag{46}$$

respectively.

Figure 3 shows dynamic stress intensity factors of a single strip for different values of $\beta^{(1)}$. It can be seen that the transient response jumps first from zero to a constant value, which corresponds to the appropriate static stress intensity factor of a cracked anisotropic infinite plane as indicated in Eq. (37), at the instance ($t = b_1 h$) that the incident wave reaches the crack tip. At time $t = b_1\sqrt{[h^2 + (2\beta^{(1)}l)^2]}$, the dynamic stress intensity factor jumps again to another constant value until $t = b_1(h + 2\beta^{(1)}l)$. Then it will approach its corresponding static value after the first nine waves have passed through the crack tip. Moreover, it is also found that the stress intensity factor

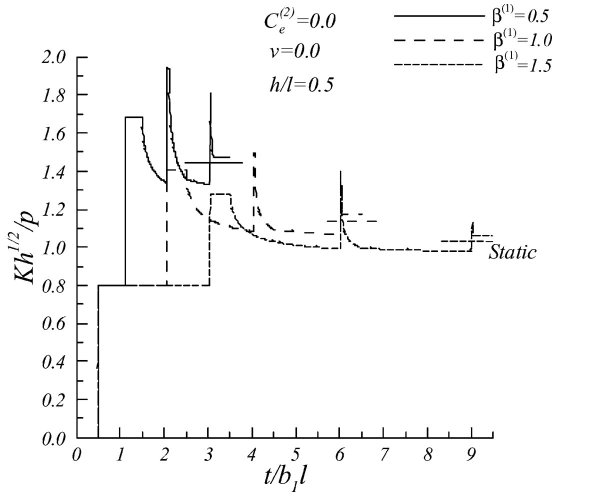

Fig. 3 Transient stress intensity factors of an anisotropic single strip for different values of $\beta^{(1)}$.

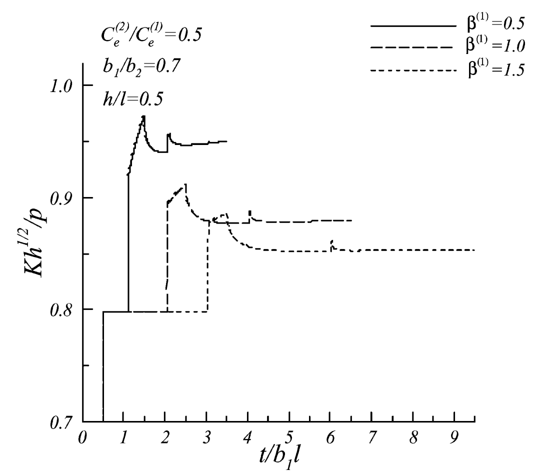

Fig. 4 Transient stress intensity factors of an anisotropic layered medium for different values of $\beta^{(1)}$ with $C_e^{(2)}/C_e^{(1)}$ = 0.5.

decreases as the value of $\beta^{(1)}$ increases for this single-strip problem. The reason is that the width of the strip after linear coordinate transformation will be widened with the increment of $\beta^{(1)}$.

The dynamic stress intensity factors of the anisotropic layered medium for various situations are shown in Figs. 4–10. Figures 4 and 5 show the transient responses for different values of $\beta^{(1)}$ with $C_e^{(2)}/C_e^{(1)} = 0.5$ and 2.0, respectively. Although there is not any static value for comparison, it is seen that the dynamic stress intensity factor tends to approach the appropriate static value after the first nine waves have passed through the crack tip. It is also found that, in the case of $C_e^{(2)}/C_e^{(1)} = 0.5$, the smaller value of $\beta^{(1)}$ will induce a larger stress intensity factor, whereas it is to the contrary in the case of $C_e^{(2)}/C_e^{(1)} = 2.0$. This indicates that the combination of elastic moduli of the anisotropic layered medium has much influence on the determination of the stress intensity factor. Besides, it is noticed in Figs. 4 and 5 that the greater the value of $\beta^{(1)}$, the longer it will take to approach the static value. Figure 6 shows the transient responses of dynamic stress intensity factors for different ratios of $C_e^{(2)}/C_e^{(1)}$, in which $C_e^{(2)}/C_e^{(1)} = 0$ corresponds to the traction-free boundary condition and $C_e^{(2)}/C_e^{(1)} = \infty$ corresponds to the rigid boundary condition. It is indicated that the traction-free boundary condition will cause the largest value of stress intensity factor in the transient analysis.

Figures 7 and 8 show dynamic stress intensity factors for different ratios of b_1/b_2 with $C_e^{(2)}/C_e^{(1)} = 0.5$ and 2.0, respectively. It can be viewed that the stress intensity factor is small when the magnitude of the ratio b_1/b_2 is small, which is independent of the ratio

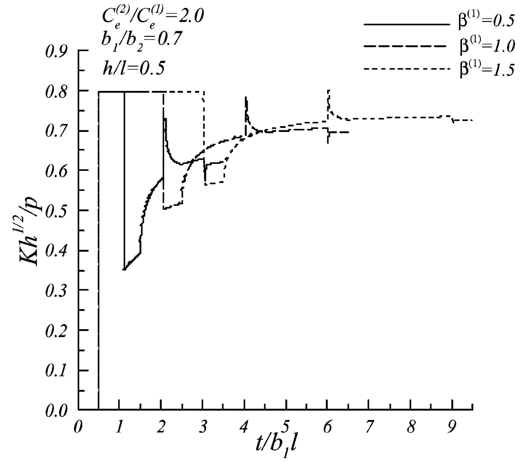

Fig. 5 Transient stress intensity factors of an anisotropic layered medium for different values of $\beta^{(1)}$ with $C_e^{(2)}/C_e^{(1)}$ = 2.0.

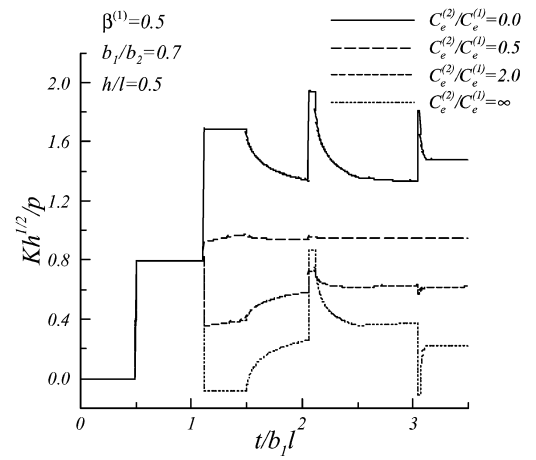

Fig. 6 Transient stress intensity factors of an anisotropic layered medium for different ratios of $C_e^{(2)}/C_e^{(1)}$.

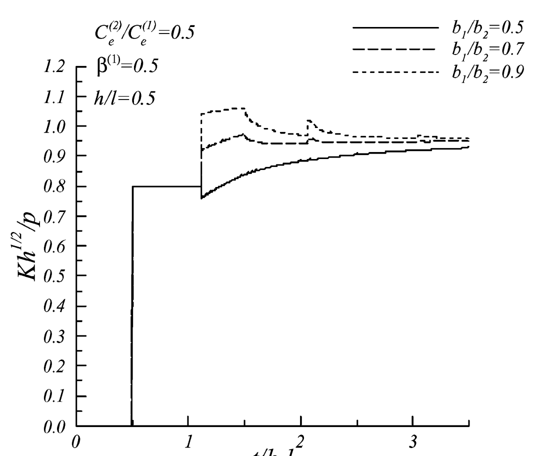

Fig. 7 Transient stress intensity factors of an anisotropic layered medium for different ratios of b_1/b_2 with $C_e^{(2)}/C_e^{(1)}$ = 0.5.

of $C_e^{(2)}/C_e^{(1)}$. Transient responses of dynamic stress intensity factors for different ratios of h/l are shown in Figs. 9 and 10 with $C_e^{(2)}/C_e^{(1)} = 0.5$ and 2.0, respectively. It is worthy to note that, in the case of $C_e^{(2)}/C_e^{(1)} = 0.5$, the stress intensity factor raises as the value of the ratio h/l increases, whereas the stress intensity factor becomes smaller with a greater value of h/l in the case of $C_e^{(2)}/C_e^{(1)} = 2.0$. Figure 10 indicates that, for a constant value of h in the case of $C_e^{(2)}/C_e^{(1)} = 2.0$, the more slender the middle strip, the smaller the stress intensity factor will be. The reason is that more energy is

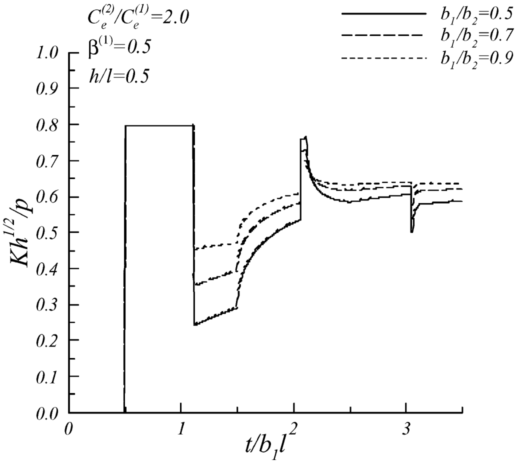

Fig. 8 Transient stress intensity factors of an anisotropic layered medium for different ratios of b_1/b_2 with $C_e^{(2)}/C_e^{(1)} = 2.0$.

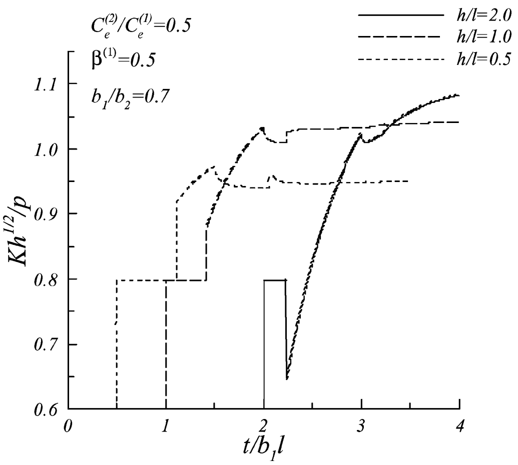

Fig. 9 Transient stress intensity factors of an anisotropic layered medium for different ratios of h/l with $C_e^{(2)}/C_e^{(1)} = 0.5$.

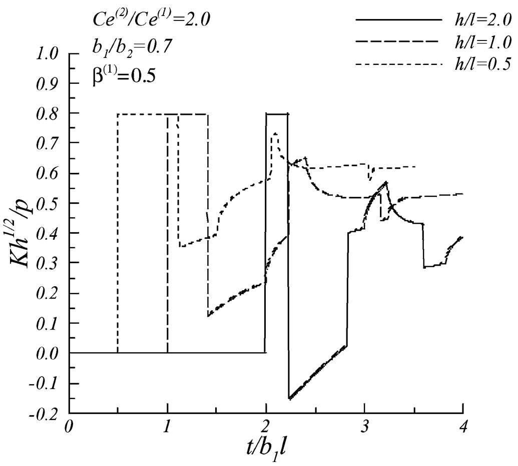

Fig. 10 Transient stress intensity factors of an anisotropic layered medium for different ratios of h/l with $C_e^{(2)}/C_e^{(1)} = 2.0$.

dissipated by the refractions for the case $C_e^{(2)}/C_e^{(1)} = 2.0$. So the combination of elastic moduli plays a significant role in dynamic fracture analysis.

Conclusions

The transient problem of an anisotropic layered medium containing a semifinite crack subjected to dynamic antiplane loading has been investigated to gain insight into the phenomenon of the interaction of stress waves between cracks and interfaces. The interaction in isotropic layered media has been discussed in several experimental and analytical works. This study extends the theoretical analysis from isotropic problems to anisotropic problems. A modified linear coordinate transformation is introduced in this study to simplify the problem. The linear coordinate transformation successfully reduces the anisotropic problem to an equivalent isotropic problem without complicating the geometric boundary conditions. This problem contains characteristic lengths and is solved by superposition of fundamental solutions in the Laplace transform domain. Explicitly transient solutions for dynamic stress intensity factors are obtained and expressed in closed form. These solutions are valid for infinitely long time and have accounted for the contribution of incident, reflected, and multiple diffracted waves. Every term in the solution has its own physical meaning. Numerical calculations are performed to investigate the transient behavior of the dynamic stress intensity factor and the results are discussed in detail. Numerical results indicate that the influence on dynamic stress intensity factors induced by reflected waves is significant and the transient effect can be neglected after the first nine waves have passed the crack tip.

Acknowledgment

The authors gratefully acknowledge the financial support of this research by the National Science Council (Republic of China) under Grant NSC 90-2212-E-032-011.

References

[1]Ma, C. C., and Ing, Y. S., "Transient Analysis of a Crack in a Composite Layered Medium Subjected to Dynamic Loadings," *AIAA Journal*, Vol. 35, No. 4, 1997, pp. 706–711.

[2]Ting, T. C. T., "Recent Developments in Anisotropic Elasticity," *International Journal of Solids and Structures*, Vol. 37, No. 1, 2000, pp. 401–409.

[3]Ting, T. C. T., *Anisotropic Elasticity: Theory and Applications*, Oxford Univ. Press, New York, 1996.

[4]Kundu, T., and Boström, A., "Elastic Wave Scattering by a Circular Crack in a Transversely Isotropic Solid," *Wave Motion*, Vol. 15, No. 3, 1992, pp. 285–300.

[5]Zhang, Ch., and Gross, D., *On Wave Propagation in Elastic Solids with Cracks*, Computational Mechanics Publ., Southampton, England, U.K., 1998.

[6]Rizza, R., and Nair, S., "A Penny Shaped Crack in a Transversely Isotropic Material Under Non-Axisymmetric Impact Loads," *International Journal of Solids and Structures*, Vol. 36, No. 1, 1999, pp. 35–64.

[7]Zhao, X., "The Stress-Intensity Factor History for a Half Plane Crack in a Transversely Isotropic Solid Due to Impact Point Loading on the Crack Faces," *International Journal of Solids and Structures*, Vol. 38, No. 16, 2001, pp. 2851–2865.

[8]Sarkar, J., Mandal, S. C., and Ghosh, M. L., "Diffraction of Elastic Waves by Three Coplanar Griffith Cracks in an Orthotropic Medium," *International Journal of Engineering Science*, Vol. 33, No. 2, 1995, pp. 163–177.

[9]Itou, S., and Haliding, H., "Dynamic Stress Intensity Factors Around Two Parallel Cracks in an Infinite-Orthotropic Plane Subjected to Incident Harmonic Stress Waves," *International Journal of Solids and Structures*, Vol. 34, No. 9, 1997, pp. 1145–1165.

[10]Das, S., and Patra, B., "Moving Griffith Crack at the Interface of Two Dissimilar Orthotropic Half Planes," *Engineering Fracture Mechanics*, Vol. 54, No. 4, 1996, pp. 523–531.

[11]Das, S., and Patra, B., "Stress Intensity Factors for Moving Interfacial Crack Between Bonded Dissimilar Fixed Orthotropic Layers," *Computers and Structures*, Vol. 69, No. 4, 1998, pp. 459–472.

[12]Kassir, M. K., and Bandyopadhyay, K. K., "Impact Response of a Cracked Orthotropic Medium," *Journal of Applied Mechanics*, Vol. 50, No. 3, 1983, pp. 630–636.

[13]Shindo, Y., and Nozaki, H., "Impact Response of a Finite Crack in an Orthotropic Strip," *Acta Mechanica*, Vol. 62, 1991, pp. 87–104

[14]Rubio-Gonzalez, C., and Mason, J. J., "Response of Finite Cracks in Orthotropic Materials Due to Concentrated Impact Shear Loads," *Journal of Applied Mechanics*, Vol. 66, No. 2, 1999, pp. 485–491.

[15]Rubio-Gonzalez, C., and Mason, J. J., "Green's Functions for the Stress Intensity Factor Evolution in Finite Cracks in Orthotropic Materials," *International Journal of Fracture*, Vol. 108, No. 4, 2001, pp. 317–335.

[16]Rubio-Gonzalez, C., and Mason, J. J., "Dynamic Stress Intensity Factors at the Tip of a Uniformly Loaded Semi-Infinite Crack in an Orthotropic Material, *Journal of Mechanics and Physics of Solids*, Vol. 48, No. 5, 2000, pp. 899–925.

[17]Brock, L. M., and Hanson, M. T., "An Exact Transient Analysis of Plane Wave Diffraction by a Crack in an Orthotropic or Transversely Isotropic Solid," *International Journal of Solids and Structures*, Vol. 39, No. 21, 2002, pp. 5393–5408.

[18]Zhang, C., "Transient Elastodynamic Antiplane Crack Analysis of Anisotropic Solids," *International Journal of Solids and Structures*, Vol. 37, No. 42, 2000, pp. 6107–6130.

[19]Boström, A., "Review of Hypersingular Integral Equation Method for Crack Scattering and Application to Modeling of Ultrasonic Nondestructive Evaluation," *Applied Mechanics Reviews*, Vol. 56, No. 4, 2003, pp. 383–405.

[20]Shen, S., and Kuang, Z. B., "Wave Scattering from an Interface Crack in Laminated Anisotropic Media," *Mechanics Research Communications*, Vol. 25, No. 5, 1998, pp. 509–517.

[21]Pramanik, R. K., Pal, S. C., and Ghosh, M. L., "Transient Response Due to a Pair of Antiplane Point Impact Loading on the Faces of a Finite Griffith Crack at the Bimaterial Interface of Anisotropic Solids," *International Journal of Engineering Science*, Vol. 36, No. 11, 1998, pp. 1197–1213.

[22]Ma, C. C., and Liao, C. Y., "Elastodynamic Analysis of Antiplane Anisotropic Interface Cracks," *Journal of Engineering Mechanics*, Vol. 125, No. 8, 1999, pp. 891–898.

[23]Ing, Y. S., and Ma, C. C., "Transient Response of a Finite Crack Subjected to Dynamic Anti-Plane Loading," *International Journal of Fracture*, Vol. 82, No. 4, 1996, pp. 345–362.

[24]Ing, Y. S., and Ma, C. C., "Dynamic Fracture Analysis of a Finite Crack Subjected to an Incident Horizontally Polarized Shear Wave," *International Journal of Solids and Structures*, Vol. 34, No. 8, 1997, pp. 895–910.

[25]Ing, Y. S., and Ma, C. C., "Exact Transient Full-Field Analysis of a Finite Crack Subjected to Dynamic Anti-Plane Concentrated Loadings in Anisotropic Materials," *Proceedings of the Royal Society of London* (submitted for publication).

[26]Ing, Y. S., and Ma, C. C., "Dynamic Fracture Analysis of Finite Cracks by Horizontally Polarized Shear Waves in Anisotropic Solids," *Journal of the Mechanics and Physics of Solids*, Vol. 51, No. 11, 2003, pp. 1987–2021.

[27]Cagniard, L., *Reflexion et Refraction des Ondes Seismiques Progressives*, Cauthiers-Villars, Paris, 1939; translated by E. A. Flinn, and C. H. Dix, *Reflection and Refraction of Progressive Seismic Waves*, McGraw–Hill, New York, 1962.

[28]Kuo, M. K., "Transient Stress Intensity Factors for a Cracked Plane Strip Under Anti-Plane Point Forces," *International Journal of Engineering Science*, Vol. 30, No. 2, 1992, pp. 199–211.

[29]Ma, C. C., "Relationship of Anisotropic and Isotropic Materials for Antiplane Problems," *AIAA Journal*, Vol. 34, No. 11, 1996, pp. 2453–2456.

[30]Lin, R. L., and Ma, C. C., "Antiplane Deformation for Anisotropic Multilayered Media by Using the Coordinate Transform Method," *Journal of Applied Mechanics*, Vol. 67, No. 3, 2000, pp. 597–605.

[31]Achenbach, J. D., *Wave Propagation in Elastic Solids*, Elsevier, New York, 1973, pp. 267–269.

[32]Sih, G. C., *Handbook of Stress Intensity Factors*, Inst. of Fracture and Solid Mechanics, Lehigh Univ., Bethlehem, PA, 1973, p. 2.4.1-1.

B. Sankar
Associate Editor